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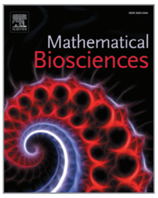

Which facilitates the evolution of cooperation more, retaliation or persistence?

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ABSTRACT

The existence of cooperation in this world is a mysterious phenomenon. One of the mechanisms that explain the evolution of cooperation is repeated interaction. If interactions between the same individuals repeat and individuals cooperate conditionally, cooperation can evolve. A previous study pointed out that if individuals have persistence (i.e., imitate its "own" behavior in the last move), cooperation can evolve. However, retaliation and persistence are not mutually exclusive decisions, but rather a trade-off in the decision making process of individuals. Players can refer to the opponent's behavior and if the actor and the opponent opted for the different alternative in the last move, conditional cooperators have to give up either retaliation or persistence. The previous study also investigated this, and has revealed that the individual should give more importance to retaliation than to persistence. However, this study has assumed that the errors in perception are absent. In this world, errors in perception are present, and trying to imitate the opponent player can sometimes end in failure. And, it might be that imitating the focal player, which definitely ends in success, is more beneficial than trying to imitate the opponent player, which can end in failure especially when the error rate in recognition is large. Here, this paper uses evolutionarily stable strategy (ESS) analysis and analyzes the stability for reactive strategies against the invasion by unconditional defectors in the iterated prisoner's dilemma game. And our analysis reveals that even if we take errors in perception into consideration, retaliation facilitates the evolution of cooperation more than persistence unexpectedly. In addition, we analyze the stability for reactive cooperators against the invasion by a strategy other than unconditional defectors. Moreover, we also analyze the deterministic model in which unconditional cooperators, unconditional defectors, and the reactive strategy at the same time.

1. Introduction

Cooperation is defined as the act which is costly to the actor and beneficial to the recipient [38]. If we consider cooperation in terms of natural selection, it is expected that cooperation will diminish as time goes by. However, cooperation is abundant in this world. We can see that there is a contradiction and this mysterious phenomenon has been a major topic in evolutionary biology [16,38,39,50].

Thus far, some mechanisms that explain the evolution of cooperation have been proposed. One of the mechanisms is direct reciprocity [1,50]. Trivers [50] mentioned that if interactions repeat and individuals have retaliation (i.e., behave cooperatively with a higher probability when the opponent cooperated in the last move than in the case wherein the opponent defected in the last move), the evolution of cooperation is facilitated in the sense of evolutionarily stability against the invasion by unconditional defectors. This is because a reciprocator can elicit future cooperation from the opponent reciprocator, while a defector cannot elicit future cooperation from the opponent reciprocator. Retaliation facilitates the evolution of cooperation [50] (but see also [8,22]).

Cooperation in repeated interactions can be studied by using the framework called the iterated prisoner's dilemma game (IPD) [1]. Assume that individuals are paired at random. Individuals choose to either cooperate or defect in each round. An individual who cooperates will give an opponent an amount b at a personal cost of c, where $b > c > 0$, while an individual who defects will give nothing. The probability that the individuals interact more than t times in a given pair is given as w^t, where $0 < w < 1$. As w increases, so does the number of interactions per pair. It is straightforward to obtain that the expected number of interactions is $1/(1-w)$.

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Relevant to direct reciprocity is imperfect information [4,19,20,22,23,25,28]. Information is sometimes absent in this world, and in such cases players cannot imitate the opponent's behavior. How should players behave when information about the opponent is unavailable?

Kurokawa [25], by using a mathematical model, examined the case wherein interactions repeat and information is sometimes absent. And Kurokawa [25] found that if individuals have persistence (i.e., behave cooperatively with a higher probability in the case wherein the actor cooperated in the last move than in the case wherein the actor defected in the last move) when information about the opponent is unavailable, the evolution of cooperation is facilitated in the sense of evolutionarily stability against the invasion by unconditional defectors.

This result can be interpreted as follows. When the conditional cooperator behaves retaliatory as well as with persistence, imitating the actor's behavior finally can lead to imitating the opponent's behavior; hence, a conditional cooperator with persistence can elicit future cooperation from the opponent cooperator. Persistence can be one type of retaliation and facilitates the evolution of cooperation. And persistence has very recently been examined theoretically and empirically [15,25].

Does having persistence when information about the opponent is present facilitate the evolution of cooperation? It might be that they think that having persistence when information about the opponent is present facilitate the evolution of cooperation since that logic is applicable also for the case wherein information is available. However, the situation wherein information about the opponent is present and the situation wherein information about the opponent is absent are qualitatively different. Let us explain the difference in the following.

If both the actor and the opponent cooperated in the last move, conditional cooperators should cooperate in the following round. If so, conditional cooperators can give importance to both retaliation and persistence. If both the actor and the opponent defected in the last move, the individual should defect in the following round. If so, conditional cooperators can give importance to both retaliation and persistence. These two cases are easy for the individual to choose a behavior.

Difficult are the following case. If the actor and the opponent opted for the different alternative in the last move, what should conditional cooperators do in the following round? If conditional cooperators give importance to retaliation, conditional cooperators have to give up persistence. Similarly, if conditional cooperators give importance to persistence, conditional cooperators have to give up retaliation. That is, retaliation and persistence are not mutually exclusive decisions, but rather a trade-off in the decision making process of individuals. Thus, what the individual should do in the following round in the case wherein the actor and the opponent opted for the different alternative in the last move is not easily determined.

Kurokawa [25] also tackled on this topic, and found that under some condition (revisit this in more detail in Model section) if individuals put more importance on retaliation than on persistence, the evolution of cooperation is facilitated.

This result can be interpreted as follows. Imitating the actor's behavior finally can lead to imitating the opponent's behavior; therefore, imitating the actor's behavior is beneficial. This is the mechanism for which imitating the actor's behavior is beneficial. Therefore, imitating the opponent's behavior directly is more beneficial than imitating the actor's behavior if players can refer not only to the opponent's behavior but also to the actor's behavior [25].

However, this study contained the following assumption. Our previous work assumed that the players always succeed in regarding cooperation by the opponent as cooperation. However, in this world, it can be considered that there exist errors in perception [2,3,18,36,37,41]. Even when taking the existence of perception errors into consideration, is referring to the opponent's behavior still more beneficial than referring to the focal player's behavior? When errors in recognition are present, since players sometimes recognize the opponent player's cooperation as defection, trying to imitate the opponent's player does not always result in succeeding in imitating the opponent player. That is, in the presence of errors in recognition, the player can defect even when the opponent player cooperated in the previous round because of errors in recognition. Thus, when errors in recognition are present, players cannot always succeed in imitating the opponent player. And it is not obvious which is more beneficial, to try to imitate the opponent player, which can end in failure, or to imitate the focal player, which definitely ends in success. Especially when the error rate in recognition is large, imitating the opponent player ends in failure with a high probability. Hence, we presume that in such a case imitating the focal player, which definitely ends in success, is more beneficial than trying to imitate the opponent player, which can end in failure with a high probability.

In this paper, we tackle on the following question: Is the statement in our previous paper "Retaliation facilitates the evolution of cooperation more than persistence" still robust when we take errors in perception into consideration?

The rest of the paper is structured as follows. In Section 2, we describe a model. In Section 3, we introduce our previous work, which assumed that errors in perception are absent. In Section 4, we consider the case where errors in perception can occur. Especially, in Section 4.1, we consider the case where an ALLD mutant invades the population consisting of strategies with a variety of persistence (or retaliation), and use evolutionarily stable strategy (ESS) analysis. And we examine how perception errors affect the evolutionary outcome. Subsequently, in Section 4.2, we consider the case where a various mutant (not limited to ALLD mutant) invades the population and examine the stability by using evolutionarily stable strategy (ESS) analysis. In Section 4.3, we consider the three strategies game, and observe dynamics. In Section 5, we summarize the result obtained in this paper, and discuss why the results are obtained.

2. Model

As seen in Introduction section, in the absence of information, the decision making is easy since people cannot behave retaliatory. On the other hand, in the presence of information, decision making is difficult since people can behave retaliatory, and retaliation and persistence are not mutually exclusive decisions. Hence, this paper focuses on the case where information about the opponent is always present.

It may be that the players do not always succeed in regarding cooperation by the opponent as cooperation. Our previous work (Section 3) considered the case where errors in perception never occur. In this paper (Section 4), we introduce a type of mistake in the iterated prisoner's dilemma game: the error in perception [2]. Reactive cooperators mistakenly regard cooperation by the opponent as defection when errors in perception occur. We use e, where $0 \le e < 1$, to denote the probability that such an error occurs. We can say that Section 3 considers the special case in which $e = 0$ holds true. On the other hand, regarding their "own" behaviors, we assume that reactive cooperators always succeed in perception for their "own" behaviors throughout this paper.

We consider the following strategy. The space of strategies for a game for the current case would be a vector of five probabilities: f, P_{CC}, P_{CD}, P_{DC}, and P_{DD}. f represents the probability of trying to cooperate in the first round. P_{ij} represents the probability of trying to cooperate when the focal player did i and the focal player recognized that the opponent did j in the last move. P_{CC}, P_{CD}, P_{DC}, and P_{DD} are not less than 0 and not more than 1 since P_{CC}, P_{CD}, P_{DC}, and P_{DD} are probabilities. And let us assume that $0 < f < 1$ holds true.

Thus, we have used the terms "retaliation" and "persistence" without strict definitions. Here, let us define the terms "retaliation" and "persistence" in terms of the $P_{i,j}$ values formally. Our previous work [25] stated that $P_{iC} - P_{iD}$ can be regarded as retaliation when the focal player did i in the last move (note that there are two kinds of retaliation since there are two behaviors for i), and that $P_{Cj} - P_{Dj}$ can be regarded as persistence when the opponent player did j in the last move (note that there are two kinds of persistence since there are two behaviors for j). Let us use these definitions following Kurokawa [25].

Now we have $(P_{CC} - P_{CD}) + (P_{CD} - P_{DD}) = P_{CC} - P_{DD} \leq 1$, which indicates that retaliation in the case where the focal player cooperated in the last move $(P_{CC} - P_{CD})$ and persistence in the case where the opponent player defected in the last move $(P_{CD} - P_{DD})$ are not mutually exclusive decisions, but rather a trade-off. Similarly, we have $(P_{CC} - P_{DC}) + (P_{DC} - P_{DD}) = P_{CC} - P_{DD} \leq 1$, which indicates that persistence in the case where the opponent player cooperated in the last move $(P_{CC} - P_{DC})$ and retaliation in the case where the focal player defected in the last move $(P_{DC} - P_{DD})$ are not mutually exclusive decisions, but rather a trade-off.

Kurokawa [25] considered the case where errors in perception are absent, and showed that the strategy whose stability condition against the invasion by unconditional defector (ALLD) mutant is the loosest among the strategies which satisfy $P_{CC} + P_{DD} = P_{CD} + P_{DC}$ is the strategy with $P_{CC} = P_{DC} = 1$ and $P_{CD} = P_{DD} = 0$ (, which does not have persistence) while a strategy with persistence can be the best among the strategies who do not have the restriction $P_{CC} + P_{DD} = P_{CD} + P_{DC}$. Since the purpose of this paper is to examine whether the result "retaliation facilitates the evolution of cooperation more than persistence" obtained in Kurokawa [25] is swayed or not by the existence of errors in perception, we only study the evolution of the strategies with the constraint $P_{CC} + P_{DD} = P_{CD} + P_{DC}$ throughout this paper (let us name the strategies with this constraint R). Note that this assumption can be re-expressed as $P_{CC} - P_{CD} = P_{DC} - P_{DD}$, which means that retaliation in the case where the focal player cooperated in the last move is equal to retaliation in the case where the focal player defected in the last move, and note that this assumption can be also re-expressed as $P_{CC} - P_{DC} = P_{CD} - P_{DD}$, which means that persistence in the case where the opponent player cooperated in the last move is equal to persistence in the case where the opponent player defected in the last move.

Under the constraint $P_{CC} - P_{CD} = P_{DC} - P_{DD}$, we have the following two relationships; one is $(P_{CC} - P_{CD}) = (P_{DC} - P_{DD})$, and the other is $(P_{CC} - P_{DC}) = (P_{CD} - P_{DD})$. These equalities mean that the degree of retaliation is independent of what the focal player did in the last move, and that the degree of persistence is independent of what the opponent player did in the last move. Thus, we can place the two kinds of retaliation in the same class (similarly, we can place the two kinds of persistence in the same class); therefore, in the following, we call *just* "retaliation" and "persistence", respectively.

All animals are error-prone; hence, we consider errors in behavior [21,33,36,47]. We use μ, where $0 < \mu < 1$, to denote the probability that mistakes in behavior occur, i.e., that an individual who intends to cooperate fails to do so and finally defects.

3. Previous studies [25]

Our previous study considered the situation wherein one single always defect (ALLD) invades a population consisting of strategy R. Our previous study showed that the condition under which the resident strategy is a strict ESS against an encroachment of ALLD is given as

$$\frac{c}{b} < \frac{(1-\mu)(P_{DC} - P_{DD})w}{1 - (P_{CD} - P_{DD})w(1-\mu)} \tag{1}$$

On one hand, as $P_{CC} - P_{CD}$ increases, the right-hand side of (1) increases. This indicates that retaliation facilitates the evolution of cooperation. On the other hand, as $P_{CD} - P_{DD}$ increases, the right-hand side of (1) increases if $P_{DC} - P_{DD} > 0$. This indicates that persistence facilitates the evolution of cooperation. However, the right-hand side of (1) is largest when $P_{CC} = P_{DC} = 1$ and $P_{CD} = P_{DD} = 0$ hold true, and this strategy does not have persistence. Retaliation and persistence are not mutually exclusive decisions, but rather a trade-off in the decision making process of individuals, and the individual should give more importance to retaliation than to persistence, this result indicates.

4. Result

4.1. Result 1

Our previous work (Section 3) examined to what probability conditional cooperators should be persistent, and revealed that the individual should give more importance to retaliation than to persistence. However, this study was based on the assumption that errors in perception are absent. In this subsection, we relax this assumption and examine the robustness of this result.

We consider the situation wherein one ALLD strategy mutant invades a population consisting of players adopting R. We define x as the expected payoff to an individual playing R for a game in a group of two Rs and define y as the expected payoff to an individual playing ALLD in a group consisting of one ALLD and one R.

Here, we have (see Appendices A and B for detailed calculation, respectively),

$$x = \frac{(1-\mu)(f - fw + P_{DD}\, w)}{(1-w)(1-(1-\mu)((P_{CC} - P_{DD}) - e(P_{DC} - P_{DD}))w)}(b-c) \tag{2}$$

$$y = \frac{(1-\mu)(f - fw + P_{DD}\, w)}{(1-w)(1-(1-\mu)(P_{CD} - P_{DD})w)}b \tag{3}$$

We can then determine the condition under which R is a strict ESS against an encroachment of ALLD. The condition is that R's payoff against itself is larger than the R's payoff against ALLD mutant, given as

$$x > y. \tag{4}$$

Using (2), (3), and (4), the condition under which R is a strict ESS against an encroachment of ALLD is given as

$$\frac{c}{b} < \frac{(1-\mu)(P_{DC} - P_{DD})(1-e)w}{1 - (P_{CD} - P_{DD})w(1-\mu)} \tag{5}$$

Substituting $e = 0$ into (5) reduces (5) to (1). Eq. (5) indicates that both persistence $(P_{CD} - P_{DD})$ and retaliation $(P_{DC} - P_{DD})$ facilitate the evolution of cooperation facilitates the evolution of cooperation. However, the right-hand side of (5) is largest when $P_{CC} = P_{DC} = 1$ and $P_{CD} = P_{DD} = 0$ hold true. The result that the individual should give more importance to retaliation than to persistence still stands even if errors in perception occur and the error rate in perception is high.

It is apparent that the right-hand side of (5) decreases as e increases, indicating that the condition under which reactive cooperators evolves becomes more stringent when more errors in perception occur. And It is also apparent that the right-hand side of

(5) decreases as μ increases. This indicates that errors in behavior will disturb the evolution of cooperation.

Eq. (5) can be re-expressed as $-c+b\sum_{t=1}^{\infty} f(t)w^t > 0$, where $f(t) = (1-e)(P_{DC}-P_{DD})(1-\mu)((P_{CD}-P_{DD})(1-\mu))^{t-1}$. Since $f(t)$ is coefficient of w^t, it is reasonable that we consider that cooperation elicits cooperation from the opponent player after t rounds with probability $f(t)$. When $P_{CD} = P_{DD}$ is met, $f(t) = 0$ holds true for any t satisfying $t > 1$, while $f(1) > 0$ holds true. This indicates that when players do not have persistence, cooperation elicits only cooperation from the opponent player in the next round, and but does not elicit cooperation from the opponent player after t ($t > 1$) round. And when $0 < P_{CD} - P_{DD} < 1$ is met, $f(t) > 0$ holds true for any t satisfying $t \geq 1$. This indicates that when players have retaliation and do refer to its own previous move, cooperation can elicit not only cooperation from the opponent player in the next round but also cooperation from the opponent player after t ($t > 1$) round. And when $P_{CD} = P_{DD} = 1$ and $P_{CD} = P_{DD} = 0$ are met, $f(t) = 0$ holds true for any t satisfying $t \geq 1$. This indicates that when players have only persistence and do not have retaliation at all, cooperation cannot elicit cooperation from the opponent player in any round.

4.2. A stability condition R when a variety of strategies invade the population of Rs

In the previous Section 4.1, we only examined the case where an invader is an unconditional defector (ALLD). In this subsection, we examine the case where a variety of mutants invade the population consisting of Rs and obtain the stability condition for R. We consider a strategy which chooses to cooperate or defect in each round as a mutant strategy in this subsection, and call the mutant strategy s. And we denote a probability that s cooperates on tth round in a game between s and R by $p(t)$. Similarly, we denote a probability that R cooperates on tth round in a game between s and R by $q(t)$. In this case, on one hand, the expected times that strategy s cooperates in a game between s and R is given as $\sum_{t=1}^{\infty} p(t)w^{t-1}$. On the other hand, the expected times that R cooperates in a game between s and R is given as $\sum_{t=1}^{\infty} q(t)w^{t-1}$. Hence, the accumulated payoff in a repeated prisoner's dilemma game to strategy s (let us define z as this) is given by

$$z = -c\sum_{t=1}^{\infty} p(t)w^{t-1} + b\sum_{t=1}^{\infty} q(t)w^{t-1}. \tag{6}$$

Here, we have

$$q(1) = f(1-\mu) \tag{7}$$

For $t > 1$, by using the restriction of R (i.e., $P_{CC}+P_{DD} = P_{CD}+P_{DC}$), we have

$$\begin{aligned} q(t) \ &= [(1-e)(P_{DC}-P_{DD})p(t-1) + (P_{CD}-P_{DD})q(t-1) + P_{DD}] \\ &\quad \times (1-\mu) \end{aligned} \tag{8}$$

Using (6)–(8), we have

$$\begin{aligned} z = {} & \frac{(1-\mu)(f - fw + P_{DD}\ w)}{(1-w)(1-(1-\mu)(P_{CD}-P_{DD})w)} b \\ & + \sum_{t=1}^{\infty} p(t)w^{t-1}\left(-c + b\frac{(1-\mu)(P_{DC}-P_{DD})(1-e)w}{1-(P_{CD}-P_{DD})w(1-\mu)}\right) \end{aligned} \tag{9}$$

Substituting $p(t) = 0$ for any t into (9), (9) reduces to $\frac{(1-\mu)(f-fw+P_{DD}\ w)}{(1-w)(1-(1-\mu)(P_{CD}-P_{DD})w)}b$, which is equivalent to y.

On the other hand, the previous Section 4.1 showed that the accumulated payoff to a R in a game between two Rs is given by (2). We can then determine the condition under which R is an ESS against an encroachment of strategy s, and the condition is given as

$$x > z. \tag{10}$$

From (2), (9), and (10), it turns out that the stability condition is given as

$$\begin{aligned} & \left(\frac{c}{b} - \frac{(1-\mu)(P_{DC}-P_{DD})(1-e)w}{1-(P_{CD}-P_{DD})w(1-\mu)}\right)\left(\sum_{t=1}^{\infty} p(t)w^{t-1}\right. \\ & \left. - \frac{(1-\mu)(f - fw + P_{DD}\ w)}{(1-w)(1-(1-\mu)((P_{CC}-P_{DD}) - e(P_{DC}-\ P_{DD}))w)}\right) > 0 \end{aligned}$$

After algebraic calculations, this becomes

$$\begin{aligned} & \left(\frac{c}{b} - \frac{(1-\mu)(P_{DC}-P_{DD})(1-e)w}{1-(P_{CD}-P_{DD})w(1-\mu)}\right) \\ & \times \left(\sum_{t=1}^{\infty} p(t)w^{t-1} - \sum_{t=1}^{\infty} q(t)w^{t-1}\right) > 0 \end{aligned} \tag{11}$$

When $\sum_{t=1}^{\infty} p(t)w^{t-1} - \sum_{t=1}^{\infty} q(t)w^{t-1} > 0$ is met (i.e., roughly speaking, an invader is more cooperative than the resident strategy), (11) becomes

$$\frac{c}{b} > \frac{(1-\mu)(P_{DC}-P_{DD})(1-e)w}{1-(P_{CD}-P_{DD})w(1-\mu)} \tag{12}$$

Conversely, when $\sum_{t=1}^{\infty} p(t)w^{t-1} - \sum_{t=1}^{\infty} q(t)w^{t-1} < 0$ is met (i.e., roughly speaking, an invader is more defective than the resident strategy), (11) becomes

$$\frac{c}{b} < \frac{(1-\mu)(P_{DC}-P_{DD})(1-e)w}{1-(P_{CD}-P_{DD})w(1-\mu)} \tag{13}$$

For example, when an invader is an ALLC (always cooperate) mutant, it can be shown that the stability condition is given as (12). On the contrary, when an invader is an ALLD mutant, the stability condition is given as (13) as shown in Section 4.1. Thus, we know that the resident strategy R is subject to an invasion by someone, irrespective of the cost-to-benefit ratio.

In the next Section 4.3, we consider the case where the three strategies (i.e., ALLC, ALLD, and R) are present, and examine what happens in the replicator dynamics.

4.3. The three strategies: ALLC (unconditional cooperation), ALLD (unconditional defection), and the reactive strategy

In the previous two Sections 4.1 and 4.2, we considered the game in pairs. In this subsection, we consider the situation where three strategies (i.e., ALLC (unconditional cooperators), ALLD (unconditional defectors), and the reactive strategy) are present at the same time.

We consider a model with ALLC, ALLD, and the reactive strategy and with payoff matrix

$$\begin{pmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{pmatrix} \tag{14}$$

Let x, y, and z denote the frequency of ALLC, ALLD, and the reactive strategy, respectively ($x+y+z=1$). And we denote F_x, F_y, and F_z the expected payoff of ALLC, ALLD, and the reactive strategy, respectively. We have

$$F_J = xa_{Jx} + ya_{Jy} + za_{Jz} \tag{15}$$

where $J = x, y, z$. And let us denote the average payoff in the population by

$$\bar{F} = xF_x + yF_y + zF_z \tag{16}$$

The change of the frequencies (i.e., evolution) of the strategies in the population can be described by the following replicator equation

$$\dot{x} = x\left(F_x - \bar{F}\right) \tag{17}$$

$$\dot{y} = y\left(F_y - \bar{F}\right) \tag{18}$$

$$\dot{z} = z\left(F_z - \bar{F}\right) \tag{19}$$

Now after algebraic calculations, we obtain a payoff matrix M. M can be described as

$$M = \begin{pmatrix} V(-c+b) & V(-c) & V(-c) + (U(1-\theta) + V\theta)b \\ Vb & 0 & U(1-\theta)b \\ -(U(1-\theta) + V\theta)c + Vb & -U(1-\theta)c & U(-c+b) \end{pmatrix} \tag{20}$$

where

$$U = \frac{(1-\mu)(f - fw + P_{DD}\ w)}{(1-w)(1-(1-\mu)((P_{CC} - P_{DD}) - e(P_{DC} - P_{DD}))w)} \tag{21}$$

$$V = (1-\mu)/(1-w) \tag{22}$$

$$\theta = \frac{(1-\mu)(P_{DC} - P_{DD})(1-e)w}{1-(P_{CD} - P_{DD})w(1-\mu)} \tag{23}$$

here, using the fact that the replicator equation remains unchanged (in the simplex S3) if we add constant to each column or we multiply each element by constant, M (which we still denote by M) can be described as (see [5,25,47] for similar works)

$$M = \begin{pmatrix} 0 & -1 & \varepsilon\sigma \\ 1 & 0 & -\rho\sigma \\ \varepsilon & -\rho & 0 \end{pmatrix} \tag{24}$$

where

$$\varepsilon = (V-U)(1-\theta)/V \tag{25}$$

$$\rho = U(1-\theta)/V \tag{26}$$

$$\sigma = \frac{b\theta - c}{c(1-\theta)} \tag{27}$$

Now, by using the domain of f, we have

$$\varepsilon > 0 \tag{28}$$

$$\rho > 0 \tag{29}$$

Firstly, let us consider the case where $\sigma < 0$. In this case, from (24), (28), and (29), it turns out that all orbits converge to $y = 1$. Unconditional defectors take over the population.

Secondly, let us consider the case where $\sigma > 0$. Let us observe the three edges (i.e., $x = 0$, $y = 0$, and $z = 0$). By using $\sigma > 0$, (24), and (29), it turns out that on the edge $x = 0$, there is a bistable competition between unconditional defectors and conditional cooperators. And there is a unstable internal equilibrium and their basins of attraction are separated by $T_{yz} = (0, 1-\hat{z}, \hat{z})$, where

$$\hat{z} = \frac{1}{1+\sigma}. \tag{30}$$

Combining (30) with (27), we also have

$$\hat{z} = \frac{c(1-\theta)}{(b-c)\theta}. \tag{31}$$

By using $\sigma > 0$, (24), and (28), it turns out that on the edge $y = 0$ there is a stable coexistence between the conditional cooperators and the unconditional cooperators. And there is a stable internal equilibrium at the point $T_{xz} = (1-\hat{z}, 0, \hat{z})$. By using (24), it turns out that on the edge $z = 0$ the unconditional defectors dominate the unconditional cooperators (see Fig. 1).

Next, let us examine the dynamics when

$$z = \hat{z} \tag{32}$$

On this line, after algebraic calculations (see Appendix C for proof), we obtain

$$\dot{x} < \dot{z} = 0 < \dot{y} \tag{33}$$

is satisfied. Hence, as times goes by, the frequency of unconditional cooperators decreases, and the frequency of unconditional defectors increases, while the frequency of conditional cooperators is independent of time (see Fig. 1).

Eq. (31) means that as θ increases, $\hat{z}$ decreases. Now, from (23), it turns out that as retaliation $(P_{DC} - P_{DD})$ increases, θ increases, and as persistence $(P_{CD} - P_{DD})$ increases, θ increases. However, retaliation $(P_{DC} - P_{DD})$ and persistence $(P_{CD} - P_{DD})$ are not independent and θ maximizes when the reactive strategy satisfies $P_{CC} = P_{DC} = 1$ and $P_{CD} = P_{DD} = 0$ (i.e., the reactive strategy is TFT), and in this case, players have only retaliation and do not have persistence. Combining these facts, we can say that $\hat{z}$ minimizes when players have retaliation but do not have persistence, which is not affected by the parameter (e).

The interior fixed point is given by

$$T = (\rho\sigma,\ \varepsilon\sigma,\ 1)\frac{1}{1+\sigma(\rho+\varepsilon)} \tag{34}$$

See also Brandt and Sigmund [5], Sigmund [47], and Kurokawa [25]. Here, by using (25)–(27), we can find that (34) can be re-expressed as

$$T = (\rho\sigma,\ \varepsilon\sigma,\ 1)\frac{c}{b\theta} \tag{35}$$

Here, we define W as

$$W = x^{\alpha} y^{\beta} z^{\gamma}[1-(1+\sigma)z]. \tag{36}$$

where

$$\alpha = \rho/\theta, \tag{37}$$

$$\beta = \varepsilon/\theta, \tag{38}$$

$$\gamma = -1/\theta. \tag{39}$$

Here, after algebraic calculations (see Appendix D for proof), we have

$$\frac{dW}{dt} = 0. \tag{40}$$

From (40), we can say that W is a constant of motion. By using this fact, it turns out that the interior fixed point T is Lyapunov stable, but not asymptotically stable. And it also turns out that periodic orbits surround the unique fixed point T (see Fig. 1).

This indicates that as θ increases, the frequency of the reactive strategy of the interior fixed point decreases. The frequency of the reactive strategy of the interior fixed point minimizes when the reactive strategy satisfies $P_{CC} = P_{DC} = 1$ and $P_{CD} = P_{DD} = 0$ (i.e., the reactive strategy is TFT, and has only retaliation but not does not have persistence), which is not affected by the parameter (e). And from (23), we can see that as the parameter (e) decreases, the parameter (θ) increases. With this fact, we can see that as the parameter (e) decreases, the frequency of the reactive strategy of the interior fixed point decreases. The frequency of the reactive strategy of the interior fixed point minimizes when $e = 0$ (i.e., errors in perception never occur).

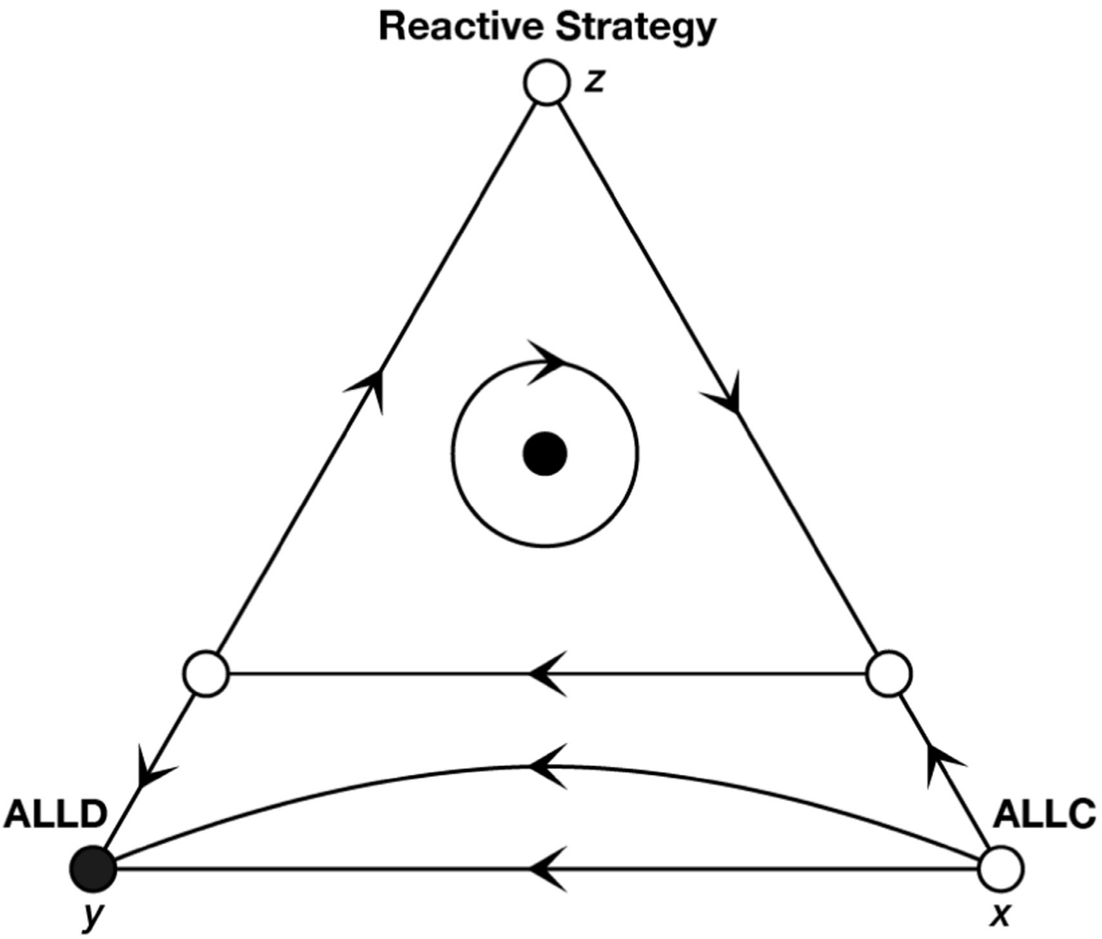

Fig. 1. The dynamics in the three strategies game when $\sigma > 0$. Fig. 1 demonstrates the dynamics in the case where $\sigma > 0$. There is a horizontal orbit on the line with $z = \hat{z}$, which connects the fixed points T_{xz} and T_{yz}. When $z = \hat{z}$ is met, $\dot{x} < 0 = \dot{z} < \dot{y}$ is satisfied. The part above the line is filled with periodic orbits which surround the unique fixed point T. Periodic orbits correspond to the constant level curves of a constant of the motion W which is given by (36). On the contrary, below this line, all orbits converge to $y = 1$, and unconditional defectors spread over the population. See Brandt & Sigmund [5] and Sigmund [47]; however, our analysis is more general.

5. Discussion

In this paper, we have examined the case wherein errors in perception are present, and have investigated the accuracy of the statement in our previous paper: Retaliation facilitates the evolution of cooperation more than persistence. In Section 4.1, we have considered the situation where an ALLD mutant invades the population. In Introduction section, we expected that it might be that the individual should give more importance to persistence which works well than to retaliation which does not work well in the case wherein such errors in perception often occur. However, unexpectedly, we have revealed that even when taking the perception errors into consideration, the statement by our previous study is robust. The individual should give more importance to retaliation than to persistence, even in the presence of errors in perception. In Section 4.2, we have examined the case where various mutants invade the population, and obtained the stability condition. Roughly speaking, the stability condition indicates that the strategies which are good at resisting the invasion by unconditional defectors are poor at resisting the invasion by unconditional cooperators. In Section 4.3, we have examined the case where unconditional cooperators and unconditional defector and the reactive strategy, and have examined the dynamics.

Let us consider why the result that retaliation facilitates the evolution of cooperation more than persistence in the case wherein errors in perception occur is obtained in Section 4.1. In general, it is required that the player cooperates for the cooperator and defects for the defector in order for cooperation to evolve. Persistence enables the player to cooperate for the cooperator and defect for the defector, since the act of the focal player reflects on the act of the opponent player and referring the previous act of the focal player (i.e., having persistence) leads to reflecting on the act of the opponent player some rounds ago. This process includes the step that the player observes the opponent's behavior, and indirectly persistence as well as retaliation is affected by errors in perception. We speculate that therefore the result that retaliation facilitates the evolution of cooperation more than persistence still stands even if errors in perception occur.

This study considers the case wherein a different player has the same benefit and the same different cost. However, the case wherein a different player has a different benefit and a different cost also seems interesting [24]. In such a case, how is the result swayed? Further study on this issue is needed.

In this paper, we assumed that the evolutionary dynamics is deterministic (e.g. [17,34]). However, examining how the stochasticity influences the evolutionary dynamics also seems fascinating [7,9,12,14,26,27,29,31,32,40,42,46,49,54,55,57,59]. Further study on this issue is required.

This study is limited to the case where the number of the players is just two. What happens in the case where over two players interact at the same time? In order to know this, extending the two player game to n-player game is necessary [6,9–11,13,26,27,29,30,35,43–45,48,51–53,56–58]. Further study on this issue is needed.

Persistence has very recently been examined theoretically and empirically [15,25], while retaliation has been examined theoretically and empirically [8,50] for decades. And retaliation and persistence are not mutually exclusive decisions in some situation. Which facilitates the evolution of cooperation more, retaliation or persistence? Further study on this interesting topic is warranted.

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Appendices

Appendix A

Proof. for (2)

Here, let A denote the total expected payoff to a strategy R in a given and subsequent rounds when both two Rs cooperate. Similarly, let B denote the total expected payoff to a strategy R in a given and subsequent rounds when the focal R cooperates and the opponent R defects. And let C denote the total expected payoff to a strategy R in a given and subsequent rounds when the focal R defects and the opponent R cooperates. And let D denote the total expected payoff to a strategy R in a given and subsequent rounds when both Rs defect. In this case, the following relationships hold:

$$\begin{aligned} x = {} & [f(1-\mu)]^2A + f(1-\mu)[1-f(1-\mu)]B \\ & + [1-f(1-\mu)]f(1-\mu)C + [1-f(1-\mu)]^2D \end{aligned} \tag{A.1}$$

$$\begin{aligned} A = {} & (b-c) + w\Bigg\{[((1-e)P_{CC}+eP_{CD})(1-\mu)]^2A \\ & +[((1-e)P_{CC}+eP_{CD})(1-\mu)] \\ & \Bigg\{1-[((1-e)P_{CC}+eP_{CD})(1-\mu)]\Bigg\}B \\ & +\Bigg\{1-[((1-e)P_{CC}+eP_{CD})(1-\mu)]\Bigg\} \\ & [((1-e)P_{CC}+eP_{CD})(1-\mu)]C \\ & +\{1-[((1-e)P_{CC}+eP_{CD})(1-\mu)]\}^2D\Bigg\} \end{aligned} \tag{A.2}$$

$$\begin{aligned} B = {} & (-c) + w\Bigg\{P_{CD}(1-\mu)[(1-e)P_{DC}+eP_{DD}](1-\mu)A \\ & +P_{CD}(1-\mu)\Bigg\{1-[(1-e)P_{DC}+eP_{DD}](1-\mu)\Bigg\}B \\ & +\Bigg\{1-P_{CD}(1-\mu)\Bigg\}[(1-e)P_{DC}+eP_{DD}](1-\mu)C \\ & +\Bigg\{1-P_{CD}(1-\mu)\Bigg\}\Bigg\{1-[(1-e)P_{DC}+eP_{DD}](1-\mu)D\Bigg\}\Bigg\} \end{aligned} \tag{A.3}$$

$$\begin{aligned} C = {} & b + w\Bigg\{[(1-e)P_{DC}+eP_{DD}](1-\mu)P_{CD}(1-\mu)A \\ & +[(1-e)P_{DC}+eP_{DD}](1-\mu)\Bigg\{1-P_{CD}(1-\mu)\Bigg\}B \\ & +\Bigg\{1-[(1-e)P_{DC}+eP_{DD}](1-\mu)\Bigg\}P_{CD}(1-\mu)C \\ & +\Bigg\{1-[(1-e)P_{DC}+eP_{DD}](1-\mu)\Bigg\}\Bigg\{1-P_{CD}(1-\mu)\Bigg\}D\Bigg\} \end{aligned} \tag{A.4}$$

$$\begin{aligned} D = {} & 0 + \Bigg\{[P_{DD}(1-\mu)]^2A + P_{DD}(1-\mu)[1-P_{DD}(1-\mu)]B \\ & +[1-P_{DD}(1-\mu)]P_{DD}(1-\mu)C + [1-P_{DD}(1-\mu)]^2D\Bigg\} \end{aligned} \tag{A.5}$$

Using (A.1)–(A.5) and $P_{CC}+P_{DD}=P_{CD}+P_{DC}$, we have (2). This is the end of the proof.

Appendix B

Proof. for (3)

Here, let P denote the total expected payoff to an ALLD mutant in a given and subsequent rounds when an ALLD mutant defects and the resident strategy cooperates. And let Q denote the total expected payoff to an ALLD mutant in a given and subsequent rounds when both an ALLD mutant and the resident strategy defect. In this case, the following three relationships hold:

$$y = Pf(1-\mu) + Q[1-f(1-\mu)] \tag{B.1}$$

$$P = b + w\{P_{CD}(1-\mu)P + (1-P_{CD}(1-\mu))Q\} \tag{B.2}$$

$$Q = 0 + w\{P_{DD}(1-\mu)P + (1-P_{DD}(1-\mu))Q\} \tag{B.3}$$

Using (B.1)–(B.3), we have (3). This is the end of the proof.

Appendix C

Proof. for (33)

On one hand, from (15), (16), and (24), we have

$$\bar{F} = (1+\sigma)zF_z \tag{C.1}$$

This simply leads to

$$F_z - \bar{F} = F_z[1-(1+\sigma)z], \tag{C.2}$$

We see that in the interior of S3, if (32) is met, then (C.2) leads to

$$F_z = \bar{F} \tag{C.3}$$

By using (19) and (C.3), we have

$$\dot{z} = 0 \tag{C.4}$$

On the other hand, by using (15) and (25)–(27), we have

$$F_x - F_y = (-1)(x+y) + (\varepsilon\sigma + \rho\sigma)z = (-1)(1-z)$$

$$+(\varepsilon\sigma + \rho\sigma)z = z(1 + \sigma(\rho + \varepsilon)) - 1 = z\frac{b\theta}{c} - 1 \tag{C.5}$$

Here, by using (27), (31) and $\sigma > 0$,

$$\hat{z}\frac{b\theta}{c} - 1 = \frac{c(1-\theta)}{(b-c)\theta}\frac{b\theta}{c} - 1 = \frac{(1-\theta)}{\left(1 - \frac{c}{b}\right)} - 1 < 0 \tag{C.6}$$

By using (C.5) and (C.6), it turns out that when (32) is met,

$$F_x - F_y < 0 \tag{C.7}$$

Here, using (16), (C.3), and (C.7), it turns out that $F_x < F_z = \bar{F} < F_y$. Here, by using (17) and (18), we know

$$\dot{x} < 0 \tag{C.8}$$

and

$$\dot{y} > 0. \tag{C.9}$$

From (C.4), (C.8), and (C.9), this is the end of the proof.

Appendix D

Proof. for (40)

From (15), (16), and (24), we have

$$\bar{F} = (1+\sigma)zF_z \tag{D.1}$$

Here, using (17)–(19), (D.1), and (36)–(39),

$$\begin{aligned}\frac{dW}{dt} &= \left(\dot{x}\frac{\alpha}{x} + \dot{y}\frac{\beta}{y} + \dot{z}\frac{\gamma}{z} + \dot{z}\frac{-(1+\sigma)}{[1-(1+\sigma)z]}\right)W \\ &= \Bigg(x\left(F_x - \bar{F}\right)\frac{\alpha}{x} + y\left(F_y - \bar{F}\right)\frac{\beta}{y} + z\left(F_z - \bar{F}\right)\frac{\gamma}{z} \\ &\quad + \bar{F}(1/(1+\sigma) - z)\frac{-(1+\sigma)}{[1-(1+\sigma)z]}\Bigg)W \\ &= \left(\alpha F_x + \beta F_y + \gamma F_z - \bar{F}(\alpha+\beta+\gamma) + \bar{F}\right)W \\ &= \left([\alpha F_x + \beta F_y + \gamma F_z] - \bar{F}[(\alpha+\beta+\gamma) - 1]\right)W = 0\end{aligned} \tag{D.2}$$

Hence,

$$\frac{dW}{dt} = 0$$

This is the end of the proof.

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